CHAPTER – II

REVIEW OF LITERATURE

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The literature related with the subject entitled “Common Fixed Points of Compatible Maps in Fuzzy Metric Spaces and Fuzzy Mathematics” is presented in this chapter has been classified into three different sections namely Fixed Points Theory and Its Application, Common Fixed Points Application for Compatible Maps and Fuzzy Metric Space and Common Fixed Point to present the research works done in the subject area in readable and synchronized format.

2.1 FIXED POINTS THEORY AND ITS APPLICATION

- Researchers have been fascinated by fixed point theory ever since Banach's famous fixed point theorem was published in 1922[1]. The literature that is now available demonstrates that Fixed Point Theory has always been an active area of study. If x = x, then a self-map of a metric space X is said to have a fixed point. In addition to serving as tools for demonstrating the existence and uniqueness of solutions for various mathematical models representing phenomena arising in various fields of study, such as steady-state temperature distribution, fluid flow, chemical equations, economic theories, epidemiology, etc., fixed point theorems are also related to the existence and properties of fixed points. It is also used to research issues with systems that are relevant to optimum control. A subfield of mathematics known as fixed point theory looks for any self-mappings in which among all the elements one of the element leftward invariant. Brouwer's fixed point theorem in 1912 served as the foundation for topological fixed point theory. Tarski's fixed point theorem from 1955 is the source of discrete fixed point theory. It is the most commonly used methodology in non-linear analysis, and the effectiveness of fixed point theory is proved in the topological and algebraic structures[2]. The availability and distinct characteristics of the fixed point theory for self-maps over the metric space could better be resolute through the changing the distances in between the points, particularly when the points are working over the control functions[3].
- The theory of metric space in mathematics uses the Banach fixed point theorem[1], sometimes referred to as the contraction mapping theorem or contraction map0ping principle. It offers a contractive approach to locate certain fixed points and ensures their existence and uniqueness for particular self-mappings of metric space. The theorem, which Stefan Banach (1892–1945) initially formulated in 1922. Functional analysis[4–7] examines linear functions that appropriately trails the

vector spaces with limit-related structures. The study of the formulation characteristics of transformations, operators between function spaces, and the spaces of functions[8–9] can be linked to the beginnings of this field. The use of integral and differential equations illustrates the significance of this field of research[10–14].

- A number of fixed point theorems in convex b-metric spaces and their applications were researched by Lili Chen[15] et al. in 2020. Research claimed that the framework of b-metric spaces showed a technique of generalising the Mann's iteration algorithm and a number of fixed point solutions. First, a convex structure is used to establish the idea of a convex b-metric space, and Mann's iteration technique is then expanded to include this space. The strong convergence theorems for two categories of contraction mappings in convex b-metric spaces are then established with the aid of Mann's iteration technique. Additionally, for the aforementioned mappings in full convex b-metric spaces, the T-stability issues of Mann's iteration process are obtained.
- Fisher[16] demonstrated the fixed point theorem using several metric spaces and an increasing function from R+ to R+. Following that, several mathematicians who study in this field use various mapping techniques, including non-expansive mappings, self-mapping, multivalued mapping, sequences of mappings, operators in Hilbert spaces, and other mapping in metric spaces, Hilbert spaces, and Banach spaces. To determine the existence and uniqueness of a solution to higher order differential and integral equations, the fixed point theorem or contraction mapping has recently been used. Fisher fixed point findings in generalised metric spaces were shown by Karim Chaira[17] et al. in 2020 using a graph. In relation to mappings defined on a generalised metric space with a graph, the fisher's fixed point theorem was discussed. The classical Fisher fixed point theorem should be viewed as being extended by this study. It expands on several previous efforts on the generalisation of metric space with graph using the Banach contraction theory. Due to its numerous applications, fixed point theory is now a particularly active topic of study. It relates to the findings that show self-mapping on a set admits a fixed point under specific circumstances. The most well-known result in metric fixed point theory is the Banach Contraction Principle.

- A fixed-point theorem for generalised weakly contractive mappings in b-metric spaces was developed by Eliyas Zinab[18] et al. in 2020. In the context of b-metric spaces, they formulated a fixed-point theorem for generalised weakly contractive mappings and came to the conclusion that a fixed point exists and is unique for self-mappings that meet the theorem.
- Khaled Berrah[19] et al.'s research on the common fixed point in complex valued b-metric space's applications and theorem was published in 2019. For four self-mappings fulfilling rational contraction, they offered a common fixed point theorem, which was shown in complex valued b-metric space. The findings of this study demonstrate that a common solution to the system of Urysohn integral equations and a system of unique solutions to linear equations both exist. Their paper's major goal was to satisfy a rational inequality on complex valued b-metric spaces by presenting common fixed point outcomes of four self-mappings.
- According to the well-known Banach contraction principle (BCP), a contraction mapping over an entire metric space has a single fixed point. Banach used the idea of a diminishing map to arrive at this significant result[20]. The development of computers and new software for rapid and efficient computing has given fixed point theory a new dimension[21]. The Brouwer fixed point theorem is essential for the numerical solution of equations. Exact quotations of the phrase "a continuous map on a close unit ball in Rn has a fixed point" are found in[22].
- Metric spaces' most generic space, which can enable one to reconsider real-world applications, plays a vital part in Real Analysis and Functional Analysis. Understanding and using the idea of topological qualities to normed linear spaces as well as metric space in many domains is always intriguing as well as demanding for mathematicians. Meir-Keeler[23] produced fixed-point solutions using weakly uniformly rigorous contraction in the context of entire metric spaces as a continuation of numerous generalisations. Additionally, there are many other uses for metric fixed point theory, including "dynamic programming, variational inequalities, fractal dynamics, dynamical systems of mathematics, and the placement of satellites in the right orbits for space science".
- In their research on various fixed point theorems in partial metric spaces with applications, Kanayo Stella Eke and Jimevwo Godwin Oghonyon[24], established a

fixed point theorem for the integral type of these maps. This study claimed that the class of generalised weakly C-contractive mappings in partial metric space and he proved some fixed point results for such maps in ordered partial metric spaces without utilising the continuity of any of the functions. The outcome generalises Chen and Zhu's findings as well as those of other authors in the literature.

- Every function F has at least one fixed point under particular circumstances, as mentioned according to fixed-point theorem. These results have been cited as some of mathematics' most important ones[25]. A fixed point will always be reached by iterating the function in question if the Banach fixed-point theorem is true. Every continuous function on the closed unit ball in n-dimensional Euclidean space has a fixed point according to the Brouwer fixed-point theorem[26]. The theory does not, however, outline how to find the fixed point. The cosine function must have a single, constant value because of its continuation. At a given point, the cosine curve of y cos intersects the plane x, y. The value of this fixed point is 0.739 085 133 215 16[27]. The number of fixed points may be determined using Lefschetz's fixed-point theorem from algebraic topology. In PDE theory, Banach's fixed-point theorem[1] has been generalised[28]. Infinite dimensions can be used to establish fixed point theorems[29]. The collage theorem in fractal compression[30] shows that the fundamental description of a function quickly converges on the desired picture when applied repeatedly to any beginning image.
- A fixed point is a solution to a nonlinear partial differential equation that does not change after the equation has been applied. Therefore, u (x, t) is a fixed point of the operator that defines it if u(x, t) is a solution of a nonlinear PDE[31]. Fixed points play a crucial role in the analysis of nonlinear PDEs because they enable us to comprehend the behaviour of solutions. If we can show that a nonlinear PDE has a unique fixed point[32], it is obvious that only one solution to an equation fulfils particular starting or boundary conditions.
- Fixed point theorems in a novel class of modular metric spaces were the focus of Duran Turkoglu et al.'s[33] research. According to them, they establish a novel idea of generalised modular metric space by taking into account both a modular metric space in the sense of jleli and samet. Then he gives some illustrations to demonstrate that there is a metric structure in the generalised modular metric space. On generalised modular metric spaces, they offered certain fixed-point conclusions

for mappings of the contraction and quasi-contraction types.

- Akkouchi[34] established a general common fixed point problem for two pairs {f, S} and {g, T} of weakly compatibles self-maps of a complete b-metric (X, d; s). These maps are satisfying a contractive condition defined by a class of implicit relations in five variables. This contraction unifies, in one go, several contractive conditions previously used in a set of recent papers dealing with fixed point or common fixed results for self-maps of b-metric spaces.

- Harmati IÁ, and Kóczy[35] mentioned that recurrent neural networks called fuzzy cognitive mapping (FCMs) are used to simulate complicated systems utilising weighted causal links. The behaviour of an iteration serves as the basis for inference about the simulated system in FCM-based decision-making. Extensions of fuzzy cognitive maps, fuzzy grey cognitive maps (FGCMs) impose ambiguous weights between the concepts. An iterative process that may converge to an equilibrium point, but may also exhibit limit cycles or chaotic behaviour, determines the inference. In research necessary settings for sigmoid FGCM fixed points' actuality and exceptionality were also very well described.

- Biley[36] discovered a common fixed point theorem for four generalised S-fuzzy metric space self-mappings. The goal of the research was to develop a broader version of the common fixed point theorem that would generalise Singh and Chauhan's finding about the idea of compatibility in fuzzy metric space.

- In order to explain the idea of fuzzy metric space in various forms, Dixit and Gupta[37] developed particular common fixed point theorems for set valued and single valued mappings in fuzzy metric space and fuzzy 2 - metric spaces. The majority of the research's findings either deal with commuting maps or presuppose weak commutativity of mappings. In common fixed point theory, existence theorems were widely proved using the concept of compatible maps. Furthermore, the property E.A in FM-space and the shared fixed points of two incompatible maps were applied. The outcome allowed for the generalisation of several significant fixed point theorems and expanded the field of research for common fixed points under contractive type circumstances.

- Rehman et al.[38] used three-self-mappings to study several coincidence point and common fixed point theorems in fuzzy metric spaces and showed the uniqueness of some of these conclusions by utilising the weak compatibility of three-self-mappings. The paper also provided some illustrative examples for the validation of the results in support of the findings. The use of fuzzy differential equations to support the work allows for the extension and improvement of several outcomes. Work produced some generalised fuzzy-contraction findings for three weakly compatible self-mappings in FM spaces without making the assumption that the "fuzzy contractive sequences[96] are Cauchy." Further, also demonstrated more coincidence points and CFP outcomes for various contractive type mappings in FM spaces by employing this idea and integral operators.

- Different applications of fixed-point theorems and metric fixed-point theory are made to find a special common solution to differential equations and integral equations. Murthi et al.[39] have demonstrated a few fixed-point theorems in the context of bipolar metric spaces using the extension of Meir-Keeler contraction. Non-trivial examples have been added to the work or the derived findings. In addition to providing an application to discover an analytical solution to an Integral Equation in order to augment the generated result, the results expanded and generalised the conclusions reached in the past.

2.2 COMMON FIXED POINTS APPLICATION FOR COMPATIBLE MAPS

- Several academicians or scholars have used specific connections in the system of fuzzy metric spaces to illustrate standard fixed point theorems. Common fixed-point theorems in generalised fuzzy metric spaces for weakly compatible mappings meeting common E.A. like characteristics. Fixed point assertions in Digital Topology is discussed by Boxer[40, 93, 94] with keeping the freezing sets fits to the theory of the digital topology in order to present the corrections associated with fixed points. The research rephrases almost all valid published statements about digital metric spaces employing the metric rather than the adjacency. Therefore, as a consequence, it appears that the digital metric space is an artificial construct with compromised concern with digital pictures. It was noticed that numerous assertions have been made in the literature on digital metric spaces are counterparts for subsections of Euclidean R^n. The authors frequently overlooked the crucial distinctions between the topological space R^n and digital pictures, leading to false or falsely "proven assertions," trivial, or inconsequential. For instance, many fixed point assertion-satisfying functions essentially be continuous or failed to be continuous digitally.

- Fixed points are strongly connected to these equations since they frequently occur as solutions to nonlinear partial differential equations (PDEs). A fixed point in mathematics is a value that remains constant while the function is applied[41]. A fixed point is a solution that doesn't change after applying the equation while discussing nonlinear partial differential equations. So, if u (x, t) is a nonlinear PDE solution, u (x, t) is a fixed point of the operator that defines it[42]. Fixed points are an important aspect of the study of nonlinear PDEs because they allow us to better understand the behaviour of solutions[43]. If we can show that a nonlinear PDE has a unique fixed point, we know that only one solution to an equation fulfils some initial or boundary conditions[44]. Furthermore, we know that an equation can have more than one solution if we can show that a nonlinear PDE has numerous fixed points. Sometimes, the existence or absence of fixed points can be used to demonstrate the existence or absence of nonlinear PDE solutions. In the analysis of nonlinear PDEs, the fixed point idea is a potent tool that is commonly employed to investigate the existence, uniqueness, and stability of solutions[45].

- Based on the idea of compatible and weakly compatible self-mappings in fuzzy cone metric spaces, research confirmed the use of a few generalised common fixed point theorems for four of these self-mappings[46]. According to research, the conclusions of[47, 48] may be generalised and extended to include self-mappings with continuity of a self-map h and without continuity for a pair of weakly compatible self-mappings.

- Ali et al.'s work[49] employed the ideas of sub-compatibility and sub-sequential continuity to demonstrate a common fixed point theorem for six self-maps in a fuzzy metric space. A number of previous fixed point results in metric space and fuzzy metric space are generalised, expanded, united, and fuzzified by the established result. In the study, a common fixed point theorem for six self-maps in a fuzzy metric space was demonstrated using the ideas of sub-compatibility and sub-sequential continuity. Several fixed point findings in metric space and fuzzy metric space have been generalised, extended, united, and fuzzified. These results may be further extended by expanding the number of self-maps with a new class of inequality. It was investigated if certain full metric spaces of mappings are generically well-posed for fixed point problems, and the very first.

- If and only if x = x is true when f is a mapping from a set or a space X into itself, a point x X is referred to as a fixed point of the mapping[50]. Theorems about fixed points are those that discuss their existence and properties[51]. The most important tools for proving the validity of the solutions to the various mathematical models (differential, integral, partial differential equations, variational inequalities, etc.) that represent various phenomena relevant to various fields, such as steady state temperature distribution, chemical reactions, neutron transport theory, economic theories, epidemics, and fluid flow, are these theorems. They are also used to investigate the problems with optimum control that occur in these systems.

- In 2017, Verma and Shrivastava[52] demonstrated reciprocal continuity for idempotent mappings in fuzzy metric spaces as well as weak commuting in fuzzy metric spaces. This chapter also establishes some conclusions on the existence and uniqueness of fixed point theorems in M-fuzzy metric spaces. Reciprocal continuity for idempotent mappings in M-fuzzy metric spaces, which is shown with examples to supplement our main thesis, has been used to demonstrate the existence of a fixed point theorem.

- The Picard-Banach contractions and some nonexpansive mappings are among the numerous contractive type mappings that belong to the broad class of enriched contractions that Berinde and Pacurar[53] presented. According to research, every enriched contraction has a singular fixed point that may be approximated using the right Krasnoselski iterative approach. The fixed points of local enriched contractions, asymptotic enriched contractions, and enriched contractions of the Maia type were also reported in the research. The research article also included examples to demonstrate the universality of new ideas and accompanying fixed point theorems.

- Under the presumptions that these two pairs of self-maps are weakly compatible and meet a contractive condition, Sekhar[54] demonstrated the presence of shared fixed points between two pairs of self-maps. A series of self-maps is added as an extension of the same. Additionally, the investigation supported the same results with various assumptions on two pairs of self-maps, one of which is weakly compatible and the other of which is compatible and reciprocally continuous. The same results with various hypotheses on two pairs of self-maps, where any of the pair satisfies the (E.A) condition and limits the completeness of X to its subspace, were likewise validated by the study. The research illustrated the extension of Babu and Dula[55] through maps presenting 2-pairs where one pair was weakly companionable. The identical approach is followed for successive self-maps.

- Common fixed point theorems in metric spaces were examined by Semwal and Komal[56] along with its certain scope applications. For specific contractive types of mappings, work examined the existence and uniqueness of common fixed point theorems followed with the advancements. Findings and results are helpful in examining the existence and distinctiveness of common solutions for a set of functional equations that arise in dynamic programming. While studying the various fixed point theorems in partial metric spaces with applications, Eke and Oghonyon[57] for the integral type of these maps established a fixed point theorem. The work claimed class of generalized weakly C-contractive mappings in partial metric space and also proved some fixed point results for such maps in ordered partial metric spaces without utilizing the continuity of any of the functions. The existence and uniqueness of shared fixed points of sometimes weakly compatible mappings meeting particular contractive requirements in a Gsymmetric space were shown by Eke and Oghonyon in 2019[58]. To attain their research goals, they worked upon E. A. Property.

- The proposed primary theorem is a generalised version of a few well-known theorems, as demonstrated by the fact that Patel and Bhardwaj[59] have proven various fixed point and common fixed point theorems for Cone metric space in integral type mappings. The fuzzy metric spaces (Fuzzy 2 and 3 metric spaces) as well as various varieties of fuzzy metric spaces are both amenable to this theorem.

2.3 FUZZY METRIC SPACE, FUZZY MATHEMATICS AND COMMON FIXED POINT

Fuzzy cognitive maps employ directed graphs with edges from the range [1, 1] with constant weights to reflect the direction and intensity of causal linkages. In the FCM theory, the nodes are referred to as "concepts" and represent certain components of the represented system. The numbers in the [0, 1] or [1, 1] interval, known as "activation values," are also used to describe the ideas' present states. In some of the results[60–61] importance of fuzzy logic is addressed for many engineering and related engineering fields, it was indicated that the processing of human perceptions and cognitions served as an inspiration for fuzzy logic, which is founded on the idea of relative graded memberships. Information derived from computational senses and cognitions, which is ambiguous, opaque, imprecise, partially true, or without distinct bounds, can be handled using fuzzy logic. The integration of hazy human judgements in computational problems is made possible by fuzzy logic. For many persons involved in inventive work, such as “engineering, mathematics, computer software, earth science, and physics”, fuzzy logic is incredibly helpful.

- In nonlinear analysis, the metric fixed point theory has been instrumental. It has often entailed the blending of topological and geometrical features. It has been extensively researched and improved upon after the renowned Banach contraction principle, either by altering the contractive condition or the underlying space. By guaranteeing the existence of fixed point, common fixed point, and coincidence point results with various types of applications, such as differential-type applications, integral-type applications, and functional-type applications, many researchers gave generalisation and improved the BCP in many directions for single-valued and multivalued mappings in the context of metric spaces. The notion of fuzzy sets was first proposed by Zadeh[62] in his foundational study, while Goguen[63] subsequently generalised fuzzy sets to L-fuzzy sets. According to the rules of fuzzy logic, certain numbers that are not part of the set are defined as elements within the range [0, 1], in contrast to conventional logic. Zadeh has been able to learn theories of fuzzy sets (FSs) that carry the problem of indefinity thanks to uncertainty, the essential component of genuine difficulty. For a variety of processes, one of which makes use of fuzzy logic, the theory is viewed as a fixed point in the fuzzy metric space (FMS). As a generalisation of fuzzy metric spaces, Park[64] created intuitionistic fuzzy metric spaces. An L-fuzzy fixed point theorem in full metric spaces was proven by Rashid et al.[65]. The fixed points of many fuzzy and L-fuzzy mappings in classical, ordered, fuzzy, and intuitionistic fuzzy metric spaces are then obtained.

- Researchers are constantly interested in learning about new discoveries in metric-space and their potential characteristics. Gahler[66] as a consequence presented the concept of 2-metric spaces, providing the idea of new dimensions for conventional metric spaces. The measure used in this context is non-negative real, or [0, +]; it has several uses. The idea of probabilistic metric spaces, which examines the probabilistic distance between two places, has given the topic and interest in knowing more about stars in the universe a new depth. Similar research was conducted on fuzzy metric spaces by Grabiec[67] and Michalek[68], which considered the degree of agreement and disagreement. The majority of the effort was clearly based on actual figures, whether they are "2-metric, fuzzy metric, modular metric, etc."
- By using fuzzy contractive mappings in non-Archiemedean fuzzy metric space, Mihet[69] established the fixed point theorem and proposed the concept. For two generalisation contractive type mappings, Vetro[70] obtained some Common fixed point solutions. The idea of intuitionistic $(\emptyset, \Psi)$ contractive mappings is explained by Abu-Doniaa et al.[71], along with certain popular fixed point theorems in intuitionistic fuzzy metric space that are proved to be true under these conditions. For compatible and weakly compatible self-mappings obeying the more generalised form of the fuzzy cone Banach contraction theorem in fuzzy cone metric spaces, Rehman et al.[46] found several common fixed point findings. With the condition of Mf triangular, research verified the generalise findings for four self-mappings both with and without a continuous self-map, h.
- The processing of human perceptions and cognitions served as the inspiration for fuzzy logic, which is founded on the idea of relative graded memberships. Information derived from computational senses and cognitions, which is ambiguous, opaque, imprecise, partially true, or without distinct bounds, can be handled using fuzzy logic. The integration of hazy human judgements in

computational issues is made possible by fuzzy logic. For many persons involved in inventive work, such as engineering (electrical, chemical, civil, environmental, mechanical, industrial, geological, etc.), mathematics, computer software, earth science, and physics, fuzzy logic is incredibly helpful. Some of their conclusions are presented in[72-74].

- Using implicit relations, Rana et al.[75] developed a few fixed-point theorems for FM-spaces. Through the use of the concepts of compatible maps, implicit relations, weakly compatible maps, and R-weakly compatible maps, several writers have described the number of fixed-point theorems in FMspaces. With the use of continuous t-norms, George and Veeramani[76] refined the idea of FM-spaces and established several fundamental features.
- Sometimes, the existence or absence of fixed points can be used to demonstrate the existence or absence of nonlinear PDE solutions. If we can show that there are no fixed points, then we know that there are no nontrivial solutions to a nonlinear PDE. In the analysis of nonlinear PDEs, the fixed point idea is a potent tool that is widely employed to investigate the existence, uniqueness, and stability of solutions[77]. For mappings meeting the -contractive condition on a fuzzy metric space, several fixed point theorems are proven that are both intuitively stated and compatible with following continuous mappings.
- In fuzzy metric space and fuzzy 2-metric spaces, Dixit and Gupta[37] demonstrated a few basic fixed point theorems for set valued and single valued mappings. The findings of the work dealt either with commuting mappings or assumed the notion of weak commutativity of mappings presented by Seesa. The concept of fuzzy metric space was introduced in many varieties. The outcome of the work allowed for the generalisation of a number of significant fixed point theorems and expanded the field of research for common fixed points under contractive type constraints. In FM-spaces, certain fixed-point theorems have been proven by Rana et al.[78] using implicit relations. In addition, fixed-point theorems have been introduced in FM-spaces utilising the concepts of compatible maps, implicit relations, weakly compatible mappings, and R-weakly compatible maps[41].
- The main goal of this study is to extend the common limit range feature that Gupta et al.[79] presented to V-fuzzy metric spaces. By using this characteristic on V -

fuzzy metric spaces, substantial results for linked maps are also demonstrated. To be more exact, we define the concept of CLR-property for the mappings $\Theta: \mathcal{M}\times\mathcal{M}\rightarrow\mathcal{M}$ and $\Omega: \mathcal{M}\rightarrow\mathcal{M}$. We present and demonstrate our new fixed point results using our new idea.

- The term "common limit in the range property," or "CLR property," in fuzzy metric spaces was defined by Sintunavarat and Kumam[98]. In contrast to property (E-A), which necessitates this requirement for the existence of the fixed point, the idea of property CLR never needs the closed-ness of the subspace. It is clear that academics are focusing on this property in order to generalise or enhance earlier findings that were supported by the idea of property (E-A). In their publication, Jha and Pant[80] generalised and refined a number of conclusions on fixed point in fuzzy metric space under this condition (E-A) by removing the continuity of mappings even the completeness. Jha and Pant[80] established several common fixed point theorems in fuzzy metric space with property (E-A).

- The exploration of fixed points for contractive mappings is a well-known problem in metric spaces, and Huang and Zhang's presentation of the cone metric space is one such generalisation[81-82]. In addition to substituting an ordered Banach space for the collection of real numbers from a metric space, they provided some crucial results for a self-map satisfying a contractive condition in this space.

- Open spheres and closed spheres are convex in convex metric spaces, and all normed spaces and their convex subsets are convex metric spaces, according to Takahashi[83], who first introduced the concept of convex metric spaces and studied the fixed-point theory for non-expansive mappings in such a setting. Convex metric spaces that are not embedded in a normed space, however, are common. Generalised metric spaces were renamed to G-metric spaces by Mustafa and Sims[84], who also discovered several topological characteristics[95].

- Particularly, Sun and Yang[85] introduced the idea of GFMS and generalised the concept of fuzzy metric spaces. A distinct common fixed point theorem for six weakly compatible mappings in G-fuzzy metric spaces was established by Balasubramanian et al. in 2016[86]. In order to demonstrate how the iteration process converges to the unique fixed point under various contractive mapping conditions on the GFMS in convex structure, a new three-step iteration procedure is developed in this study effort. The examination into the data dependency on the outcomes of these iterative processes in the generalised G-fuzzy convex metric spaces is also a major emphasis of this chapter. To get fixed point and common fixed point theorems for a pair of self-mappings under suitable contractive type requirements with convex structure, the idea of convex structure in GFMS has also been proposed. Extensions of fuzzy cognitive maps known as fuzzy grey cognitive maps (FGCMs) were developed for the situation when only hazy information was available on the connections between the various components of the system[87]. The iteration may reach a fixed point, enter a limit cycle, or exhibit chaotic patterns, just like the classical FCMs. As a result, the fixed points issue is also very important in the context of FGCMs.

- Pazhani[88] studied "fixed point theorems from fuzzy metric spaces and intuitionistic fuzzy metric spaces" and remarked that one of the most effective and successful nonlinear analysis methods is the Fixed Point Theory, which may be viewed as the nonlinear analysis's core. The adjective "fuzzy" has gained a lot of popularity and usage in recent research pertaining to the logical and set-theoretical underpinnings of mathematics. In a number of application domains, the idea of defining Intuitionistic Fuzzy Sets (IFS) for fuzzy set generalisations has shown to be fascinating and helpful. Research has opened up new possibilities for advancing the theory of intuitionistic fuzzy metric spaces (IFMS) in a path that may be interesting for other branches of mathematics and computer science as well as general topology. With the use of the t-norm, Agnihotri et al.[89] developed the idea of fuzzy metric space. Through the use of weak compatibility, we developed a common fixed point theorem for seven self-mappings in fuzzy metric space. On intuitionistic fuzzy metric space, Abu-Donia et al.[90] established a few often used coupled fixed point theorems for mappings under the ψ-contractive condition under compatible and subsequently continuous mappings.

- A common fixed point theorem for six self-maps in a fuzzy metric space was established by Ali et al.[91] combining the ideas of sub-compatibility and sub-sequential continuity. A number of previous fixed point results in metric space and fuzzy metric space are generalised, expanded, united, and fuzzified by the established result.

- In order to realize the weak compatible mappings (wc-mapping), Vijayalakshmi et al.[92] demonstrated a fixed-point technique on E -Fuzzy-metric Space, offering Joint Common Limit in the Range (JCLR)-property implicitly. Key conclusions from the research were shown with several specific cases. For six finite families of self-mappings, work demonstrated a fixed-point theorem that may be used to support additional conclusions. Furthermore, it was demonstrated that typical fixed-point theorems may be proved using any finite number of mappings[97]. Analysis and applied mathematics both heavily rely on metrics that are typically thought of as functions of distance. But in other situations, like as the determination of the distance between two pixels in image analysis, which is frequently regarded as two-pixel similarity, metrics based on the Crisp notion are not appropriate. Therefore, based on fuzzy notions, Lukman Zicky et al.[99] established the idea of fuzzy metric. Then, convergence and fixed point issues were addressed using this fuzzy metric. Some characteristics of regular metric still hold true for fuzzy metric thanks to work.

- Hasan[100] made the most of the variable distance function by identifying some general fixed point theorems for tangential mappings for hybrid couples of both sorts of mappings (single and multi-valued). A number of earlier recognised discoveries were expanded upon and generalised by these theorems. The amount of conclusion generalisation and the veracity of assumptions were both checked by the author.

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