COMMON FIXED POINTS OF COMPATIBLE MAPS IN FUZZY METRIC SPACES AND FUZZY MATHEMATICS

An

Abstract

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ABSTRACT

This Ph.D. thesis, titled "Common Fixed Points of Compatible Maps in Fuzzy Metric Spaces and Fuzzy Mathematics," submitted to the Faculty of Mathematics at Pacific Academy of Higher Education and Research University, Udaipur, Rajasthan, explores the intersection of fuzzy metric spaces and fixed point theory. The study begins with an overview of metric spaces, introducing the concept of fuzziness and its application in mathematics. Fuzzy metric spaces, defined using fuzzy scalars, provide a unique perspective that extends the classical metric spaces, opening avenues for research in fuzzy optimization and pattern recognition.

The thesis establishes the consistency of the fuzzy topology induced by fuzzy metric spaces with prescribed topologies and introduces the notion of a compatible pair of mutually continuous mappings. This leads to a fixed point theorem in fuzzy metric spaces, with a distinctive feature that it does not necessitate mapping continuity. The exploration further extends to compatible mappings in generalized fuzzy metric spaces, introducing common fixed point theorems. Semi-compatibility and weak compatibility concepts are introduced, offering alternative conditions for mapping continuity.

The research objectives revolve around obtaining fixed point and common fixed point theorems for incompatible maps, fuzzy metric spaces, and compatible maps in fuzzy metric spaces. The work is organized into five chapters, covering general introductions, a review of literature, fuzzy metric spaces, fixed point theorems in compatible mapping, and a concluding chapter with future research directions.

The literature review presents an overview of fixed points theory, common fixed points application for compatible maps, and the role of fuzzy metric spaces in common fixed points. Chapter 3 discusses fuzzy metric space, presenting definitions, properties, and mathematical notations, along with basics of fuzzy logic relevant to fuzzy metric spaces. Chapter 4 centers on fixed point theorems in compatible mapping, examining various types and their descriptions in the context of compatible maps. The chapter concludes with outcomes and insights into fixed point theorems across different spaces. Chapter 5 summarizes findings, concludes the research work, and outlines potential future directions. References are organized alphabetically,

citing authors and publication years, with research papers included in reputed journals and filed at the thesis's end.

The research concludes with an exploration of the application of fuzzy set theory in engineering, showcasing its impact across various disciplines. Fuzzy set theory's applications extend to neural network theory, stability theory, mathematical programming, medical sciences, image processing, and more, significantly advancing fields such as civil, electrical, mechanical, robotics, industrial, computer, and nuclear engineering.

Fuzzy set theory contributes to fixed and common fixed point theorems in fuzzy metric spaces, broadening its applications to topology and analysis. The versatility of fuzzy metric spaces transcends mathematical domains, finding applications in engineering and quantum particle physics. Its ability to model uncertainty in real-world scenarios demonstrates its invaluable role in addressing complex and uncertain problems across diverse disciplines. As research continues to unfold the theory of fuzzy sets and their applications, the impact is expected to grow, offering innovative solutions to theoretical and practical challenges.

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