COMMON FIXED POINTS OF COMPATIBLE MAPS IN FUZZY METRIC SPACES AND FUZZY MATHEMATICS

A

Thesis

Submitted for the Award of the Ph.D. degree of

PACIFIC ACADEMY OF HIGHER EDUCATION AND RESEARCH UNIVERSITY

By

SHEFAL HARSHADSINH VAGHELA

Under the Supervision of

Dr. RITU KHANNA
Professor
Department of Mathematics
Faculty of Engineering, Pacific University,
Udaipur (Rajasthan), India.

Dr. SHAILESH T. PATEL
Professor
S. P. B. Patel Engineering College, Linch,
(Mehsana), Gujarat

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

PACIFIC ACADEMY OF HIGHER EDUCATION AND RESEARCH UNIVERSITY UDAIPUR

DECLARATION

I, **SHEFAL HARSHADSINH VAGHELA** D/o **SHRI HARSHADSINH VAGHELA** resident of Rajvansh, Plot No. 117/2, Sector 7, C, Near Shopping Center, Gandhinagar, Gujarat – 382007, hereby declare that the research work incorporated in the present thesis entitled **"COMMON FIXED POINTS OF COMPATIBLE MAPS IN FUZZY METRIC SPACES AND FUZZY MATHEMATICS"** is my own work and is original. This work (in-part or in full) has not been submitted to any University for the award of a Degree or a Diploma. I have properly acknowledged the material collected from secondary sources wherever required. I solely own the responsibility for the originality of the entire content.

Date: / /2024

(SHEFAL HARSHADSINH VAGHELA)

Signature of the Candidate

DEPARTMENT OF MATHEMATICS

PAHER UNIVERSITY, UDAIPUR

Dr. RITU KHANNA
Professor

CERTIFICATE

Its gives me immense pleasure in certifying that the thesis entitled **"COMMON FIXED POINTS OF COMPATIBLE MAPS IN FUZZY METRIC SPACES AND FUZZY MATHEMATICS"** and submitted by **SHEFAL HARSHADSINH VAGHELA** is based on the work research carried out under my guidance. She has completed the following requirements as per Ph.D. regulations of the University;

(i) Course work as per University rules.

(ii) Residential requirements of the University.

(iii) Regularly submitted Half Yearly Progress Report.

(iv) Published/accepted minimum of two research paper in a refereed research journal.

I recommend the submission of thesis.

Date:

(Dr. RITU KHANNA)
Supervisor

DEPARTMENT OF MATHEMATICS

S. P. B. PATEL ENGINEERING COLLEGE,

LINCH, (MEHSANA), GUJARAT

Dr. SHAILESH T. PATEL
Professor

CERTIFICATE

Its gives me immense pleasure in certifying that the thesis entitled **"COMMON FIXED POINTS OF COMPATIBLE MAPS IN FUZZY METRIC SPACES AND FUZZY MATHEMATICS"** and submitted by **SHEFAL HARSHADSINH VAGHELA** is based on the work research carried out under my guidance. She has completed the following requirements as per Ph.D. regulations of the University;

(i) Course work as per University rules.

(ii) Residential requirements of the University.

(iii) Regularly submitted Half Yearly Progress Report.

(iv) Published/accepted minimum of two research paper in a refereed research journal.

I recommend the submission of thesis.

Date:

Dr. SHAILESH T. PATEL
Co-Supervisor

COPYRIGHT

I, **SHEFAL HARSHADSINH VAGHELA,** hereby declare that the Pacific Academy of Higher Education and Research University, Udaipur Rajasthan, shall have the right to preserve, use and disseminate the thesis entitled **"COMMON FIXED POINTS OF COMPATIBLE MAPS IN FUZZY METRIC SPACES AND FUZZY MATHEMATICS"** in print or electronic format for academic research.

Date: / /2024

(SHEFAL HARSHADSINH VAGHELA)

Signature of the Candidate

ACKNOWLEDGEMENT

At this juncture of my life and career, I would like to showcase gratitude and remember everyone who have supported me in the journey of doctoral research and career building.

I sincerely thank the almighty for providing all the strength to conduct the present research. Without blessings of almighty, I could not have reached to this stage.

I am highly indebted to my Supervisor **Dr. RITU KHANNA,** Professor, Department of Mathematics, PAHER University, Udaipur for being constant support, motivation and valuable inputs encouraged me to pursue with my research. I express my deep and sincere gratitude for her valuable guidance, immense help and time devotion at every step, without which this work would not have been possible. I extend my thanks to her for being continuous source of inspiration throughout my research work.

I show gratitude towards my Co-supervisor, **Dr. SHAILESH T. PATEL**, Professor, S. P. B. Patel Engineering College, Linch, (Mehsana), Gujarat for constant guidance, support, encouragement and motivation. Without his, my research work could not have been completed. I am sincerely thankful to my guide, the faculty members of computer science dept at PAHER University and all the university staff members.

I am thankful to the University authorities especially **Prof. (Dr.) HEMANT KOTHARI**, Dean, PAHER University, Udaipur, and others in PAHER for their support and encouragement. I take this opportunity to thank all the individuals who have directly or indirectly lent their helping hand during the preparation of this dissertation.

I want to thank my parents **Mr. VAGHELA HARSHADSINH JANAKSINH** & **Mrs. VAGHELA INDUBEN HARSHADSINH** for their advices, help and untiring motivation. I express my thanks to my brother **BHUMIRAJSINH** for support and valuable support. This research is dedicated to my husband **KARNAVSINH RAJPUT** for helping me collecting data and keeping my spirit high during the process and my son **LAKSHYARAJSINH RAJPUT** for allowing me time to research and write. I would also thank many folds to my in-laws **VARSHABA RAJPUT** and **BALRAJSINH RAJPUT** for their love, understanding and prayers without which this could not had been possible.

Last but not the least, I have no words to thank my family members. I remember my all family members and all friends at this juncture.

Thank you everyone who has directly or indirectly helped me in this beautiful voyage.

(SHEFAL HARSHADSINH VAGHELA)

PREFACE

The Thesis entitled “Common fixed points of Compatible maps in fuzzy metric spaces and Fuzzy Mathematics” is submitted for the award of Ph. D degree in Faculty of Mathematics to Pacific Academy of Higher Education and Research University, Udaipur, Rajasthan. The proposed study is the carried out under the valuable guidance and supervision of Dr. Ritu Khanna, Professor & Dr. Shailesh T. Patel by Shefal H. Vaghela.

In the realm of mathematics, a metric space designates a collection of elements in which distances between any two members of this collection are established. These distances, collectively referred to as a metric, define the structure of the space. The most recognizable instance of a metric space is the three-dimensional Euclidean space. In essence, a "metric" serves as a generalization of the Euclidean metric, encompassing the essential characteristics of the Euclidean distance. In the Euclidean metric, the distance between two points is the length of the straight line connecting them. Noteworthy examples of other metric spaces arise in contexts such as elliptic geometry and hyperbolic geometry. In these scenarios, distance measurement on a sphere using angles functions as a metric, and in special relativity, the hyperboloid model of hyperbolic geometry serves as a metric space for velocities. The presence of a metric within a space gives rise to topological attributes like open and closed sets, which in turn contribute to the examination of more abstract topological spaces.

The research report deals mainly with Common fixed points of compatible maps in fuzzy metric spaces and fuzzy Mathematics. Fuzzy metric space is parts of topological space.

Fixed point theory stands as a cornerstone in the advancement of mathematics due to its fundamental role in the applications across various mathematical disciplines. A prominent tool within this domain is the Banach contraction principle, which serves as an efficient and easily discernible instrument for exploration. In this context, fuzzy metric spaces undergo a redefinition that distinguishes them from their predecessors by employing fuzzy scalars instead of fuzzy or real numbers to define the fuzzy metric. Demonstrably, any standard metric space can give rise to a complete fuzzy

metric space whenever the original space does. Moreover, the consistency of the fuzzy topology induced by the fuzzy metric spaces introduced in this study with the prescribed topology is established. These findings establish foundational elements for research endeavors in fuzzy optimization and pattern recognition. Notably, the concept of a compatible pair of mutually continuous mappings is defined, leading to a fixed point theorem in fuzzy metric spaces. This theorem yields a fixed point while not mandating continuity of the mapping. Building upon this, the notion of a compatible mapping is extended within the realm of fuzzy metric spaces. Generalized fuzzy metric spaces introduce the concept of compatibility, resulting in common fixed point theorems for compatible mappings. The investigation also delves into the concepts of semi-compatibility and weak compatibility in the context of fuzzy metric spaces, leveraging these concepts to establish a common fixed point theorem. This work enhances the conditions for mapping continuity by substituting compatibility with semi-compatibility and weak compatibility.

The research work was based on more applications on Common fixed points of compatible maps in fuzzy metric spaces and fuzzy Mathematics. The Research work basically carried around following research objectives:

- Some fixed point and common fixed point theorems for in compatible maps will be obtained.
- Some fixed point and common fixed point theorems in fuzzy metric spaces will be proved.
- Some common fixed point theorems in compatible maps in fuzzy metric spaces will be obtained.
- Some fixed point and common fixed point theorems for in fuzzy mathematics will be obtained.

The whole work included in the Thesis is divided into five different chapters:

Chapter 1 is of general introduction. The content of the chapter includes, general introduction on the fixed point theorem, fuzzy metric space and fuzzy mathematics. This chapter also describes fuzzy metric space and theorems which were used in this

work. This chapter demonstrated different compatibility mappings and their types and the methodology.

Chapter 2 is of review of literature which have presented literature associated with Fixed Points Theory and Its Application, Common Fixed Points Application for Compatible Maps and Fuzzy Metric Space and Common Fixed Point.

Chapter 3 is of Fuzzy Metric Space discussed about the Definition and Basic Properties, Formal mathematical notation definition of fuzzy metric spaces followed with the key properties and characteristics. Chapter also presented basics of fuzzy Logic relevant to fuzzy metric spaces.

Chapter 4 is of fixed point theorem in compatible mapping which describes various types of fixed point theorems, followed with the description of the theorems in context to compatible maps namely Banach's Fixed Point Theorem, Kannan's Fixed Point Theorem, Browder's Fixed Point Theorem, Rosenberg-Kannan Fixed Point Theorem and Chatterjea's Fixed Point Theorem. This chapter also describes various fixed point theorems in different spaces followed with the description of main outcome.

Chapter 5 is of Conclusion, Summary and Future Research. The chapter demonstrated various finding followed with the conclusion of the research work. This chapter also presented the possible future directions of our proposed research work.

References have been indicated in Thesis by the name(s) of the author(s) with year of publication and listed author wise in alphabetical order at the end. The main points in the chapter have been numbered in such a way that the first number indicates chapter: second number the serial order. The research papers incorporated in the Thesis have been published in reputed Journals and filed at the end of the thesis.

Research concluded that, the application of fuzzy set theory in the field of engineering has significantly impacted various disciplines and brought about new methodological possibilities. Fuzzy set theory finds applications in a wide range of applied sciences, including neural network theory, stability theory, mathematical programming, modelling theory, medical sciences, image processing, control theory, communication, and more. Its influence spans across all engineering disciplines,

including civil, electrical, mechanical, robotics, industrial, computer, and nuclear engineering, leading to advancements and improvements in these fields.

Fuzzy set theory has led to the development of fixed and common fixed point theorems that satisfy diverse contractive conditions in fuzzy metric spaces. This has extended the application of fuzzy sets to topology and analysis, allowing for the exploration of various theoretical aspects and practical implications.

The concept of fuzzy metric spaces has found numerous applications not only in mathematics but also in engineering and even in branches of quantum particle physics. Its versatility is evident in its ability to model uncertainty and vagueness in various real-world scenarios, enabling more accurate and flexible representations. Its applications have proven invaluable in addressing complex and uncertain problems across diverse disciplines, demonstrating the broad-reaching impact of this mathematical concept. As research continues to expand the theory of fuzzy sets and its applications, it is likely that its influence will continue to grow, offering innovative solutions to challenges in both theoretical and practical realms.

❐❐❐

ABSTRACT

This Ph.D. thesis, titled "Common Fixed Points of Compatible Maps in Fuzzy Metric Spaces and Fuzzy Mathematics," submitted to the Faculty of Mathematics at Pacific Academy of Higher Education and Research University, Udaipur, Rajasthan, explores the intersection of fuzzy metric spaces and fixed point theory. The study begins with an overview of metric spaces, introducing the concept of fuzziness and its application in mathematics. Fuzzy metric spaces, defined using fuzzy scalars, provide a unique perspective that extends the classical metric spaces, opening avenues for research in fuzzy optimization and pattern recognition.

The thesis establishes the consistency of the fuzzy topology induced by fuzzy metric spaces with prescribed topologies and introduces the notion of a compatible pair of mutually continuous mappings. This leads to a fixed point theorem in fuzzy metric spaces, with a distinctive feature that it does not necessitate mapping continuity. The exploration further extends to compatible mappings in generalized fuzzy metric spaces, introducing common fixed point theorems. Semi-compatibility and weak compatibility concepts are introduced, offering alternative conditions for mapping continuity.

The research objectives revolve around obtaining fixed point and common fixed point theorems for incompatible maps, fuzzy metric spaces, and compatible maps in fuzzy metric spaces. The work is organized into five chapters, covering general introductions, a review of literature, fuzzy metric spaces, fixed point theorems in compatible mapping, and a concluding chapter with future research directions.

The literature review presents an overview of fixed points theory, common fixed points application for compatible maps, and the role of fuzzy metric spaces in common fixed points. Chapter 3 discusses fuzzy metric space, presenting definitions, properties, and mathematical notations, along with basics of fuzzy logic relevant to fuzzy metric spaces. Chapter 4 centers on fixed point theorems in compatible mapping, examining various types and their descriptions in the context of compatible maps. The chapter concludes with outcomes and insights into fixed point theorems across different spaces. Chapter 5 summarizes findings, concludes the research work, and outlines potential future directions. References are organized alphabetically,

citing authors and publication years, with research papers included in reputed journals and filed at the thesis's end.

The research concludes with an exploration of the application of fuzzy set theory in engineering, showcasing its impact across various disciplines. Fuzzy set theory's applications extend to neural network theory, stability theory, mathematical programming, medical sciences, image processing, and more, significantly advancing fields such as civil, electrical, mechanical, robotics, industrial, computer, and nuclear engineering.

Fuzzy set theory contributes to fixed and common fixed point theorems in fuzzy metric spaces, broadening its applications to topology and analysis. The versatility of fuzzy metric spaces transcends mathematical domains, finding applications in engineering and quantum particle physics. Its ability to model uncertainty in real-world scenarios demonstrates its invaluable role in addressing complex and uncertain problems across diverse disciplines. As research continues to unfold the theory of fuzzy sets and their applications, the impact is expected to grow, offering innovative solutions to theoretical and practical challenges.

❐❐❐

CONTENTS